Micromechanical Analysis of a Lamina

Key Terms

Volume fraction
Weight (mass) fraction
Density
Void volume fraction
Void content
Elastic moduli
Array packing
Halphin–Tsai equations
Elasticity models
Transversely isotropic fibers
Strength
ASTM standards
Failure modes
IITRI compression test
Shear test
Coefficient of thermal expansion
Coefficient of moisture expansion

Exercise Set

3.1 The weight fraction of glass in a glass/epoxy composite is 0.8. If the specific gravity of glass and epoxy is 2.5 and 1.2, respectively, find the

1. Fiber and matrix volume fractions
2. Density of the composite

3.2 A hybrid lamina uses glass and graphite fibers in a matrix of epoxy for its construction. The fiber volume fractions of glass and graphite are 40 and 20%, respectively. The specific gravity of glass, graphite, and epoxy is 2.6, 1.8, and 1.2, respectively. Find

1. Mass fractions
2. Density of the composite

3.3 The acid digestion test left 2.595 g of fiber from a composite specimen weighing 3.697 g. The composite specimen weighs 1.636 g in water. If the specific gravity of the fiber and matrix is 2.5 and 1.2, respectively, find the

1. Theoretical volume fraction of fiber and matrix
2. Theoretical density of composite

3. Experimental density
4. Weight fraction of fiber and matrix
5. Void fraction

3.4 A resin hybrid lamina is made by reinforcing graphite fibers in two matrices: resin A and resin B. The fiber weight fraction is 40%; for resin A and resin B, the weight fraction is 30% each. If the specific gravity of graphite, resin A, and resin B is 1.2, 2.6, and 1.7, respectively, find

1. Fiber volume fraction
2. Density of composite

3.5 Find the elastic moduli of a glass/epoxy unidirectional lamina with 40% fiber volume fraction. Use the properties of glass and epoxy from Table 3.3 and Table 3.4, respectively.

3.6 Show that

$$G_{12} = \frac{G_m}{1 - V_f}$$

if the fibers are much stiffer than the matrix — that is, $G_f >> G_m$.

3.7 Assume that fibers in a unidirectional lamina are circularly shaped and in a square array. Calculate the ratio of fiber diameter to fiber center-to-center spacing ratio in terms of the fiber volume fraction.

3.8 Circular graphite fibers of 10 μm diameter are packed in a hexagonal array in an epoxy matrix. The fiber weight fraction is 50%. Find the fiber-to-fiber spacing between the centers of the fibers. The density of graphite fibers is 1800 kg/m^3 and epoxy is 1200 kg/m^3.

3.9 Find the elastic moduli for problem 3.5 using Halphin–Tsai equations. Assume that the fibers are circularly shaped and are in a square array. Compare your results with those of problem 3.5.

3.10 A unidirectional glass/epoxy lamina with a fiber volume fraction of 70% is replaced by a graphite/epoxy lamina with the same longitudinal Young's modulus. Find the fiber volume fraction required in the graphite/epoxy lamina. Use properties of glass, graphite, and epoxy from Table 3.1 and Table 3.2.

3.11 Sometimes, the properties of a fiber are determined from the measured properties of a unidirectional lamina. As an example, find the experimentally determined value of the Poisson's ratio of an isotropic fiber from the following measured properties of a unidirectional lamina:

1. Major Poisson's ratio of composite = 0.27
2. Poisson's ratio of the matrix = 0.35
3. Fiber volume fraction = 0.65

3.12 Using elasticity model equations, find the elastic moduli of a glass/epoxy unidirectional lamina with 40% fiber volume fraction. Use the properties of glass and epoxy from Table 3.3 and Table 3.4, respectively. Compare your results with those obtained by using the strength of materials approach and the Halphin–Tsai approach. Assume that the fibers are circularly shaped and are in a square array for the Halphin–Tsai approach.

3.13 A measure of degree of orthotropy of a material is given by the ratio of the longitudinal to transverse Young's modulus. Given the properties of glass, graphite, and epoxy from Table 3.1 and Table 3.2 and using the mechanics of materials approach to find the longitudinal and transverse Young's modulus, find the fiber volume fraction at which the degree of orthotropy is maximum for graphite/epoxy and glass/epoxy unidirectional laminae.

3.14 What are three common modes of failure of a unidirectional composite subjected to longitudinal tensile load?

3.15 Do high fiber volume fractions increase the transverse strength of a unidirectional lamina?

3.16 Find the five strength parameters of a unidirectional glass/epoxy lamina with 40% fiber volume fraction. Use the properties of glass and epoxy from Table 3.3 and Table 3.4.

3.17 A rod is designed to carry a uniaxial tensile load of 1400 N with a factor of safety of two. The designer has two options for the materials: steel or 66% fiber volume fraction graphite/epoxy. Use the properties of graphite and epoxy from Table 3.1 and Table 3.2. Assume the following properties for steel:

- Young's modulus of steel = 210 GPa
- Poisson's ratio of steel = 0.3
- Tensile strength of steel = 450 MPa
- Specific gravity of steel = 7.8

The cost of graphite/epoxy is five times that of steel by weight. List your material of choice if the criterion depends on just

1. Mass
2. Cost

3.18 Find the coefficients of thermal expansion for a 60% unidirectional glass/epoxy lamina with a 60% fiber volume fraction. Use properties of glass and epoxy from Table 3.3 and Table 3.4, respectively.

3.19 If one plots the transverse coefficient of thermal expansion, α_2, as a function of fiber volume fraction, V_f, for a unidirectional glass/epoxy lamina, $\alpha_2 > \alpha_m$ for a certain fiber volume fraction. Find this range of fiber volume fraction. Use properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

3.20 Find the fiber volume fraction for which the unidirectional glass/epoxy lamina transverse thermal expansion coefficient is a maximum. Use properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

3.21 Prove[31] that it is possible to have the transverse coefficient of thermal expansion of a unidirectional lamina greater than the coefficient of thermal expansion of the matrix ($\alpha_2 > \alpha_m$) only if

$$\frac{E_f}{E_m} > \frac{1+\nu_f}{\nu_m} \text{ or } \frac{E_f}{E_m} < \frac{1+\nu_f}{1+\nu_m}$$

3.22 The coefficient of thermal expansion perpendicular to the fibers of a unidirectional glass/epoxy lamina is given as 28 μm/m/°C. Use the properties of glass and epoxy from Table 3.3 and Table 3.4 to find the coefficient of thermal expansion of the unidirectional glass/epoxy lamina in the direction parallel to the fibers.

3.23 There are large excursions of temperature in space and thus composites with zero or near zero thermal expansion coefficients are attractive. Find the volume fraction of the graphite fibers for which the thermal expansion coefficient is zero in the longitudinal direction of a graphite/epoxy unidirectional lamina. Use all the properties of graphite and epoxy from Table 3.1 and Table 3.2, respectively, but assume that the longitudinal coefficient of thermal expansion of graphite fiber is -1.3×10^{-6} m/m/°C.

3.24 Find the coefficients of moisture expansion of a glass/epoxy lamina with 40% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

3.25 Assume a 60% fiber volume fraction glass/epoxy lamina of cuboid dimensions 25 cm (along the fibers) × 10 cm × 0.125 mm. Epoxy absorbs water as much as 8% of its weight. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively, and find

1. Maximum mass of water that the specimen can absorb
2. Change in volume of the lamina if the maximum possible water is absorbed

Assume that the coefficient of moisture expansion through the thickness is the same as the coefficient of moisture expansion in the transverse direction and that the glass fibers absorb no moisture.