Exercise Set

2.1 Write the number of independent elastic constants for three-dimensional anisotropic, monoclinic, orthotropic, transversely isotropic, and isotropic materials.

2.2 The engineering constants for an orthotropic material are found to be

$$E_1 = 4\,Msi,\ E_3 = 3\,Msi,\ E_3 = 3.1\,Msi,$$

$$\nu_{12} = 0.2,\ \nu_{23} = 0.4,\ \nu_{31} = 0.6,$$

$$G_{12} = 6\,Msi,\ G_{23} = 7\,Msi,\ G_{31} = 2\,Msi$$

Find the stiffness matrix [C] and the compliance matrix [S] for the preceding orthotropic material.

2.3 Consider an orthotropic material with the stiffness matrix given by

$$[C] = \begin{bmatrix} -0.67308 & -1.8269 & -1.0577 & 0 & 0 & 0 \\ -1.8269 & -0.67308 & -1.4423 & 0 & 0 & 0 \\ -1.0577 & -1.4423 & 0.48077 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \end{bmatrix} GPa$$

Find:

1. The stresses in the principal directions of symmetry if the strains in the principal directions of symmetry at a point in the material are $\varepsilon_1 = 1\ \mu m/m$, $\varepsilon_2 = 3\ \mu m/m$, $\varepsilon_3 = 2\ \mu m/m$; $\gamma_{23} = 0$, $\gamma_{31} = 5\ \mu m/m$, $\gamma_{12} = 6\ \mu m/m$
2. The compliance matrix [S]
3. The engineering constants E_1, E_2, E_3, ν_{12}, ν_{23}, ν_{31}, G_{12}, G_{23}, G_{31}
4. The strain energy per unit volume at the point where strains are given in part (1.)

2.4 Reduce the monoclinic stress–strain relationships to those of an orthotropic material.

2.5 Show the difference between monoclinic and orthotropic materials by applying normal stress in principal directions and shear stress in principal planes, one at a time and studying the resulting nonzero and zero strains.

2.6 Write down the compliance matrix of a transversely isotropic material (where 2–3 is the plane of isotropy) in terms of the following engineering constants:

E is the Young's modulus in the plane of isotropy 2–3

E' is the Young's modulus in direction 1 that is perpendicular to plane of isotropy 2–3

ν is the Poisson's ratio in the plane of isotropy 2–3

ν' is the Poisson's ratio in the 1–2 plane

G' is the shear modulus in the 1–2 plane

2.7 Find the relationship between the engineering constants of a three-dimensional orthotropic material and its compliance matrix.

2.8 What are the values of stiffness matrix elements C_{11} and C_{12} in terms of the Young's modulus and Poisson's ratio for an isotropic material?

2.9 Are ν_{12} and ν_{21} independent of each other for a unidirectional orthotropic lamina?

2.10 Find the reduced stiffness [Q] and the compliance [S] matrices for a unidirectional lamina of boron/epoxy. Use the properties of a unidirectional boron/epoxy lamina from Table 2.1.

2.11 Find the strains in the 1–2 coordinate system (local axes) in a unidirectional boron/epoxy lamina, if the stresses in the 1–2 coordinate system applied to are $\sigma_1 = 4$ MPa, $\sigma_2 = 2$ MPa, and $\tau_{12} = -3$ MPa. Use the properties of a unidirectional boron/epoxy lamina from Table 2.1.

2.12. Write the reduced stiffness and the compliance matrix for an isotropic lamina.

2.13 Show that for an orthotropic material $Q_{11} \neq C_{11}$. Explain why. Also, show $Q_{66} = C_{66}$. Explain why.

2.14 Consider a unidirectional continuous fiber composite. Start from $[\sigma] = [Q]\,[\varepsilon]$ and follow the procedure in Section 2.4.3 to get

$$E_1 = Q_{11} - \frac{Q_{12}^2}{Q_{22}} \qquad \nu_{12} = \frac{Q_{12}}{Q_{22}}$$

$$E_2 = Q_{22} - \frac{Q_{12}^2}{Q_{11}} \qquad \nu_{21} = \frac{Q_{12}}{Q_{11}} \qquad G_{12} = Q_{66}.$$

2.15 The reduced stiffness matrix [Q] is given for a unidirectional lamina is given as follows:

$$[Q] = \begin{bmatrix} 5.681 & 0.3164 & 0 \\ 0.3164 & 1.217 & 0 \\ 0 & 0 & 0.6006 \end{bmatrix} Msi \ .$$

What are the four engineering constants, E_1, E_2, ν_{12}, and G_{12}, of the lamina?

2.16 The stresses in the global axes of a 30° ply are given as $\sigma_x = 4$ MPa, $\sigma_y = 2$ MPa, and $\tau_{xy} = -3$ MPa. Find the stresses in the local axes. Are the stresses in the local axes independent of elastic moduli? Why or why not?

2.17 The strains in the global axes of a 30° ply are given as $\varepsilon_x = 4$ µin./in., $\varepsilon_y = 2$ µin./in., and $\gamma_{xy} = -3$ µin./in. Find the strains in the local axes. Are the strains independent of material properties? Why or why not?

2.18 Find the transformed reduced stiffness matrix $[\bar{Q}]$ and transformed compliance matrix $[\bar{S}]$ for a 60° angle lamina of a boron/epoxy lamina. Use the properties of a unidirectional boron/epoxy lamina from Table 2.1.

2.19 What is the relationship between the elements of the transformed compliance matrix $[\bar{S}]$ for a 0 and 90° lamina?

2.20 For a 60° angle lamina of boron/epoxy under stresses in global axes as $\sigma_x = 4$ MPa, $\sigma_y = 2$ MPa, and $\tau_{xy} = -3$ MPa, and using the properties of a unidirectional boron/epoxy lamina from Table 2.1, find the following

1. Global strains
2. Local stresses and strains
3. Principal normal stresses and principal normal strains
4. Maximum shear stress and maximum shear strain

2.21 An angle glass/epoxy lamina is subjected to a shear stress $\tau_{xy} = 0.4$ ksi in the global axes resulting in a shear strain $\gamma_{xy} = 468.3$ µin./in. in the global axes. What is the angle of the ply? Use the properties of unidirectional glass/epoxy lamina from Table 2.2.

2.22 Find the six engineering constants for a 60° boron/epoxy lamina. Use the properties of unidirectional boron/epoxy lamina from Table 2.2.

2.23 A bidirectional woven composite ply may yield equal longitudinal and transverse Young's modulus but is still orthotropic. Determine the angles of the ply for which the shear modulus (G_{xy}) are maximum and minimum. Also find these maximum and minimum values. Given: $E_1 = 69$ GPa, $E_2 = 69$ GPa, $\nu_{12} = 0.3$, $G_{12} = 20$ GPa.

2.24 A strain gage measures normal strain in a component. Experiments[12] suggest that errors due to strain gage misalignment are more appreciable for angle plies of composite materials than isotropic materials.

1. Take a graphite/epoxy angle ply of 8° under a uniaxial stress, $\sigma_x = 4$ Msi. Estimate the strain, ε_x, as measured by a strain gage aligned in the x-direction. Now, if the strain gage is misaligned by +3° to the x-axis, estimate the measured strain. Find the percentage of error due to misalignment. Use properties of unidirectional graphite/epoxy lamina from Table 2.2.
2. Take an aluminum layer under a uniaxial stress, $\sigma_x = 4$ Msi. Estimate the strain, ε_x, as measured by a strain gage in the x-direction. Now, if the strain gage is misaligned by +3° to the x-axis, estimate the measured strain. Find the percentage of error due to misalignment. Assume $E = 10$ Msi, $v = 0.3$ for aluminum.

2.25 A uniaxial load is applied to a 10° ply. The linear stress–strain curve along the line of load is related as $\sigma_x = 123\varepsilon_x$, where the stress is measured in GPa and strain in m/m. Given $E_1 = 180$ GPa, $E_2 = 10$ GPa and $\nu_{12} = 0.25$, find the value of (1) shear modulus, G_{12};and (2) modulus E_x for a 60° ply.

2.26 The tensile modulus of a 0°, 90°, and 45° graphite/epoxy ply is measured as follows to give $E_1 = 26.25$ Msi, $E_2 = 1.494$ Msi, $E_x = 2.427$ Msi for the 45° ply, respectively.

1. What is the value E_x for a 30° ply?
2. Can you calculate the values of ν_{12} and G_{12} from the previous three measured values of elastic moduli?

2.27 Can the value of the modulus, E_x, of an angle lamina be less than both the longitudinal and transverse Young's modulus of a unidirectional lamina?

2.28 Can the value of the modulus, E_x, of an angle lamina be greater than both the longitudinal and transverse Young's modulus of a unidirectional lamina?

2.29 Is the ν_{xy} for a lamina maximum for a 45° boron/epoxy ply? Use properties of unidirectional boron/epoxy lamina from Table 2.2.

2.30 In finding the value of the Young's modulus, E_x, for an angle ply, length-to-width (L/W) ratio of the specimen affects the measured value of E_x. The Young's modulus E_x^1 for a finite length-to-width ratio specimen is related to the Young's modulus, E_x, for an infinite length-to-width ratio specimen by[5]

$$E_x^1 = \frac{E_x}{1-\zeta},$$

where

$$\zeta = \frac{1}{\bar{S}_{11}}\left[\frac{3\bar{S}_{16}^2}{3\bar{S}_{66} + 2\bar{S}_{11}(L/W)^2}\right].$$

Tabulate the values of ζ for $L/W = 2, 8, 16$, and 64 for a 30° glass/epoxy. Use properties of unidirectional glass/epoxy lamina from Table 2.2.

2.31 Starting from the expression for the reduced stiffness element

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4) ,$$

derive the expression

$$\bar{Q}_{66} = \frac{1}{2}(U_1 - U_4) - U_3 \operatorname{Cos} 4\theta .$$

2.32 Initial stress–strain data are given for a uniaxial tensile test of a 45° angle ply. Find the in-plane shear modulus of the unidirectional lamina, G_{12}. Use linear regression analysis for finding slopes of curves.

σ_x (KPa)	ε_x (%)	$-\varepsilon_y$ (%)
210	0.1	0.08
413	0.2	0.16
644	0.3	0.25
847	0.4	0.33
1092	0.5	0.42

If similar data were given for a 35° angle ply, would it be sufficient to find the in-plane shear modulus of the unidirectional lamina, G_{12}?

2.33 Calculate the four stiffness invariants, U_1, U_2, U_3, and U_4, and the four compliance invariants V_1, V_2, V_3, and V_4, for a boron/epoxy lamina. Use the properties of a unidirectional boron/epoxy lamina from Table 2.2.

2.34 Show that $\bar{Q}_{11} + \bar{Q}_{22} + \bar{Q}_{12} + \bar{Q}_{66}$ is not a function of the angle of ply.

2.35 Find the off-axis shear strength and mode of failure of a 60° boron/epoxy lamina. Use the properties of a unidirectional boron/epoxy lamina from Table 2.1. Apply the maximum stress failure, maximum strain, Tsai–Hill, and Tsai–Wu failure theories.

2.36 Give one advantage of the maximum stress failure theory over the Tsai–Wu failure theory.

2.37 Give one advantage of the Tsai–Wu failure theory over the maximum stress failure theory.

2.38 Find the maximum biaxial stress, $\sigma_x = -\sigma$, $\sigma_y = -\sigma$, $\sigma > 0$, that one can apply to a 60° lamina of graphite/epoxy. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1. Use maximum strain and Tsai–Wu failure theories.

2.39 Using Mohr's circle, show why the maximum shear stress that can be applied to angle laminae differs with the shear stress sign. Take a 45° graphite/epoxy lamina as an example. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1.

2.40 Reduce the Tsai–Wu failure theory for an isotropic material with equal ultimate tensile and compressive strengths and a shear strength that is 40% of the ultimate tensile strength.

2.41 An off-axis test is used to find the value of H_{12} for use in the Tsai–Wu failure theory for a boron/epoxy system. The five lamina strengths of a unidirectional boron/epoxy system are given as follows:

$$(\sigma_1^T)_{ult} = 188 \text{ ksi}, \ (\sigma_1^C)_{ult} = 361 \text{ ksi}, \ (\sigma_2^T)_{ult} = 9 \text{ ksi}, \ (\sigma_2^C)_{ult} = 45 \text{ ksi},$$
$$\text{and } (\tau_{12})_{ult} = 10 \text{ ksi}.$$

A 15° specimen fails at a uniaxial load of 33.546 ksi. Find the value of H_{12}. Does it satisfy the inequality $H_{12}^2 < H_{11}H_{22}$, which is a stability criterion for Tsai–Wu failure theory that says failure surfaces intercept all stress axes and form a closed geometric surface[13]?

2.42 Give the units for the coefficient of thermal expansion in the USCS and SI systems.

2.43 Find the free-expansional strains of a glass/epoxy unidirectional lamina under a temperature change of –100°C and a moisture absorption of 0.002 kg/kg. Also find the temperature change for which the transverse expansional strains vanish for a moisture absorption of 0.002 kg/kg. Use the properties of a unidirectional glass/epoxy lamina from Table 2.1.

2.44 Find the coefficients of thermal expansion of a 60° glass/epoxy lamina. Use the properties of unidirectional glass/epoxy lamina from Table 2.2.

2.45 Give the units for coefficient of moisture expansion in the USCS and SI systems.

2.46 Find the coefficients of moisture expansion of a 60° glass/epoxy lamina. Use the properties of unidirectional glass/epoxy lamina from Table 2.1.