Exercise Set

4.1 Condense the following expanded laminate codes:

1. [0/45/–45/90]
2. [0/45/–45/–45/45/0]
3. [0/90/60/60/90/0]
4. [0/45/60/45/0]
5. [45/–45/45/–45/–45/45/–45/45]

4.2 Expand the following laminate codes:

1. $[45/-45]_s$
2. $[45/-45_2/90]_s$
3. $[45/0]_{3s}$
4. $[45/\pm 30]_2$
5. $[45/\pm 30]_2$

4.3 A laminate of 0.015 in. thickness under a complex load gives the following midplane strains and curvatures:

$$\begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \hline \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 2\times10^{-6} \\ 3\times10^{-6} \\ 4\times10^{-6} \\ \hline 1.2\times10^{-4} \\ 1.5\times10^{-4} \\ 2.6\times10^{-4} \end{bmatrix} \begin{matrix} in./in. \\ \\ 1/in. \end{matrix}$$

Find the global strains at the top, middle and bottom surface of the laminate.

4.4 Do global strains vary linearly through the thickness of a laminate?

4.5 Do global stresses vary linearly through the thickness of a laminate?

4.6 The global strains at the top surface of a $[0/45/60]_s$ laminate are given as

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1.686\times10^{-8} \\ -6.500\times10^{-8} \\ -2.143\times10^{-7} \end{bmatrix}$$

and the midplane strains in this laminate are given as

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 8.388\times10^{-6} \\ 4.762\times10^{-4} \\ -3.129\times10^{-3} \end{bmatrix}.$$

What are the midplane curvatures in this laminate, if each ply is 0.005 in. thick?

4.7 The global stresses in a three-ply laminate are given at the top and bottom surface of each ply. Each ply is 0.005 in. thick. Find the resultant forces and moments on the laminate if it has a top cross-section of 4 in. × 4 in.

	σ_{xx} (psi)	
Ply no.	Top	Bottom
1	-3.547×10^4	-2.983×10^3
2	-9.267×10^3	1.658×10^4
3	7.201×10^3	2.435×10^4

Ply no.	σ_{yy} (psi)	
	Top	Bottom
1	-2.425×10^4	-7.087×10^3
2	-1.638×10^4	9.432×10^3
3	3.155×10^3	3.553×10^4

Ply no.	τ_{xy} (psi)	
	Top	Bottom
1	-2.946×10^4	-5.564×10^3
2	-1.299×10^4	1.317×10^4
3	-5.703×10^3	2.954×10^4

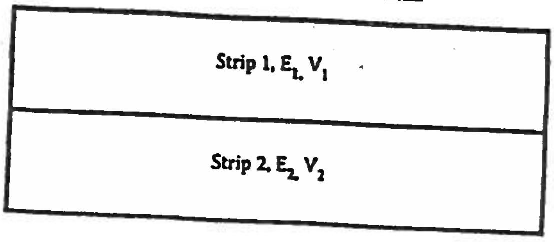

FIGURE 4.8
Laminate made of two isotropic plies.

4.8 Find the three stiffness matrices [A], [B], and [D] for a [0/60/–60] glass/epoxy laminate. Use the properties of glass/epoxy unidirectional lamina from Table 2.2 and assume the lamina thickness to be 0.005 in. Also, find the mass of the laminate if the top surface area of the laminate is 5 in. × 7 in. Use densities of glass and epoxy from Table 3.3 and Table 3.4, respectively.

4.9 Give expressions for the stiffness matrices [A], [B], and [D] for an isotropic material in terms of its thickness, t, Young's modulus, E, and Poisson's ratio, ν.

4.10 Show that, for a symmetric laminate, the coupling stiffness matrix is equal to zero.

4.11 A beam is made of two bonded isotropic strips as shown in the Figure 4.8. The two strips are of equal thickness. Find the stiffness matrices [A], [B], and [D].

4.12 Rewrite the expressions for the stiffness matrices [A], [B], and [D] in terms of the transformed reduced stiffness matrix elements, thickness of each ply, and the location of the middle of each ply with respect to the midplane of the laminate. This is called the parallel axis theorem for the laminate stiffness matrices.

4.13 Find the local stresses at the top of the 60° ply in a [0/60/–60] graphite/epoxy laminate subjected to a bending moment of $M_x = 50$ N-m/m. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1 and assume the lamina thickness to be 0.125 mm. What is the percentage of the bending moment load taken by each of the three plies?

4.14 Find the forces and moments required in a [0/60/–60] graphite/epoxy laminate to result in bending curvature of $\kappa_x = 0.1$ in.$^{-1}$ and $\kappa_y = 0.1$ in.$^{-1}$. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.2 and assume the lamina thickness to be 0.005 in.

4.15 Find the extensional and flexural engineering elastic moduli of a [45/–45], graphite/epoxy laminate. Verify the reciprocal relationships for the Poisson's ratios. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1.

4.16 Find the residual stresses at the top of the 60° ply in a [0/60/–60] graphite/epoxy laminate subjected to a temperature change of –150°F. Each lamina is 0.005 in. thick; use the properties of a unidirectional graphite/epoxy lamina from Table 2.2.

4.17 For a $[0/45]_s$ glass/epoxy laminate, find the coefficients of the thermal expansion. Use the properties of a unidirectional glass/epoxy lamina from Table 2.1. Assume thickness of each lamina as 0.125 mm. Also, find the change in the volume of the laminate if the cross-sectional area is 100 mm × 50 mm and the temperature change is 100°C.

4.18 Find the coefficients of moisture expansion of a $[0/45]_s$ graphite/epoxy laminate. The properties of a unidirectional graphite/epoxy lamina are given in Table 2.1. Assume thickness of each lamina as 0.125 mm.

4.19 Find the local stresses at the middle of the 30° ply in a [30/45] glass/epoxy laminate that is subjected to the following mechanical and hygrothermal loads: $N_x = 10^8$ lb/in.; $\Delta T = -100$°F; $\Delta C = 5\%$. Use the properties of a unidirectional glass/epoxy lamina given in Table 2.2. The thickness of each lamina is 0.005 m.

4.20 Find the difference between the vertical deflection (through the thickness) at the center and the four corners of a [0/60] graphite/epoxy cuboid laminate. The thickness of each ply is 0.005 in. and the top surface dimensions of the laminate are 20 in. × 10 in. The laminate is subjected to a temperature change of –75°F. Use the properties of a unidirectional graphite/epoxy lamina given in Table 2.2.