The following steady shear data has been obtained for a fire fighting foam at 340 kPa.

D = 6.95 mm		D = 9.9 mm		D = 15.8 mm	
τ_w (Pa)	$\frac{8V}{D}$ (s^{-1})	τ_w (Pa)	$\frac{8V}{D}$ (s^{-1})	τ_w (Pa)	$\frac{8V}{D}$ (s^{-1})
35.62	742.8	25.57	254.6	14.55	63.63
47.96	1016	29.29	348.2	18.92	87.01
58.03	1304	38.73	446.9	24.84	111.7
72.98	1617	47.91	554.3	31.54	138.5
78.88	1925	51.14	660	37.85	168
102.2	2648	60.57	907.6	35.48	165
115.7	3340	65.29	1145	42.18	226.8
		70.25	1367	44.95	286.1
		77.95	1583	48.50	341.5
				12.18	55.58
				17.54	83.42
				22.79	111.7
				29.53	139.4

The length of each capillary is 1400 mm. Obtain the true shear stress – shear rate data after making all the corrections for this foam. (Data courtesy Dr. B.S. Gardiner, University of Melbourne, Melbourne, Australia).

3.1 A low molecular weight polymer melt, which can be modelled as a power-law fluid with m = 5 k Pa s^n and n = 0.25, is pumped through a 13-mm inside diameter tube over a distance of 10 m under laminar flow conditions. Another pipe is needed to pump the same material over a distance of 20 m at the same flow rate and with the same frictional pressure loss. Calculate the required diameter of the new pipe.

3.2 The flow behaviour of a tomato sauce follows the power-law model, with n = 0.50 and m = 12 Pa s^n. Calculate the pressure drop per metre length of pipe if it is pumped at the rate of 1000 kg/h through a 25-mm diameter pipe. The sauce has a density of 1130 kg/m^3. For a pump efficiency of 50%, estimate the required power for a 50-m long pipe.
How will the pressure gradient change if

(a) the flow rate is increased by 50%,
(b) the flow behaviour consistency coefficient increases to 14.75 Pa s^n without altering the value of n, due to changes in the composition of the sauce,
(c) the pipe diameter is doubled,
(d) the pipe diameter is halved.
Is the flow still streamline in this pipe?

3.3 A vertical tube whose lower end is sealed by a moveable plate is filled with a visco-plastic material having a yield stress of 20 Pa and density 1100 kg/m^3. Estimate the minimum tube diameter for this material to flow under its own weight when the plate is removed. Does the depth of the material in the tube have any influence on the initiation of flow?

3.4 A power-law fluid ($m = 5\,\text{Pa s}^n$ and $n = 0.5$) of density $1200\,\text{kg/m}^3$ flows down an inclined plane at 30° to the horizontal. Calculate the volumetric flow rate per unit width if the fluid film is 6-mm thick. Assume laminar flow conditions.

3.5 A Bingham plastic material is flowing under streamline conditions in a circular pipe. What are the conditions for one-third of the total flow to be within the central plug region across which the velocity profile is flat? The shear stress acting within the fluid τ varies with the velocity gradient dV_z/dr according to the relation:

$$\tau = \tau_0^B + \mu_B\left(-\frac{dV_z}{dr}\right)$$

where τ_0^B and μ_B are, respectively, the Bingham yield stress and the plastic viscosity of the material.

3.6 Tomato purée of density $1100\,\text{kg/m}^3$ is pumped through a 50-mm diameter pipeline at a flow rate of $1\,\text{m}^3/\text{h}$. It is suggested that, in order to double the rate of production,

(a) a similar line with pump should be put in parallel with the existing one, or
(b) a larger pump should be used to force the material through the present line, or
(c) the cross-sectional area of the pipe should be doubled.

The flow behaviour of the tomato puree can be described by the Casson equation (1.19), i.e.,

$$(+\tau_{rz})^{1/2} = (+\tau_0^C)^{1/2} + \left[\mu_C\left(-\frac{dV_z}{dr}\right)\right]^{1/2}$$

where τ_0^C, the Casson yield stress, is equal to 20 Pa and μ_C, the Casson plastic viscosity, has a value of 5 Pa s.

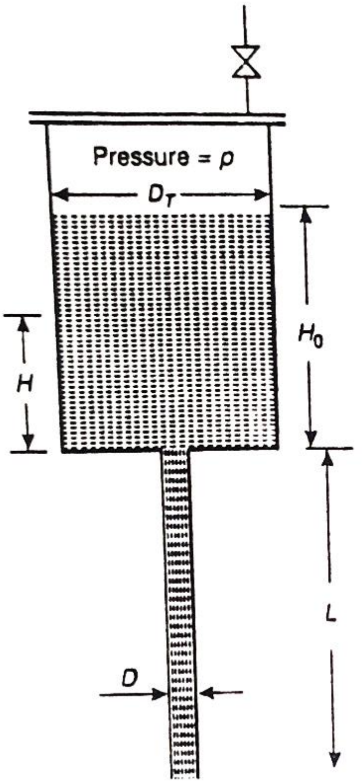
Evaluate the pressure gradient for the three cases. Also, evaluate the viscosity of a hypothetical Newtonian fluid for which the pressure gradient would be the same. Assume streamline flow under all conditions.

3.7 A polymer solution is to be pumped at a rate of 11 kg/min through a 25-mm inside diameter pipe. The solution behaves as a power-law fluid with $n = 0.5$ and has an apparent viscosity of 63 m Pa s at a shear rate of $10\,\text{s}^{-1}$, and a density of $950\,\text{kg/m}^3$.

(a) What is the pressure gradient in the pipe line?
(b) Estimate the shear rate and the apparent viscosity of the solution at the pipe wall?
(c) If the fluid were Newtonian, with a viscosity equal to the apparent viscosity at the wall as calculated in (b), what would be the pressure gradient?
(d) Calculate the Reynolds numbers for the polymer solution and for the hypothetical Newtonian fluid.

3.8 A concentrated coal slurry (density $1043\,\text{kg/m}^3$) is to be pumped through a 25-mm inside diameter pipe over a distance of 50 m. The flow characteristics of this slurry are not fully known, but the following preliminary information is available on its flow through a smaller tube, 4 mm in diameter and 1-m long. At a flow rate of $0.0018\,\text{m}^3/\text{h}$, the pressure drop across the tube is 6.9 kPa, and at a flow rate of $0.018\,\text{m}^3/\text{h}$, the pressure drop is 10.35 kPa. Evaluate the power-law constants from the data for the small diameter tube. Estimate the pressure drop in the 25-mm diameter pipe for a flow rate of 0.45 m^3/h.

3.9 A straight vertical tube of diameter D and length L is attached to the bottom of a large cylindrical vessel of diameter D_T ($\gg D$). Derive an expression for the time required for the liquid height in the large vessel to decrease from its initial value of $H_0(\ll L)$ to $H(\ll L)$ as shown in the following sketch.



Neglect the entrance and exit effects in the tube as well as the changes in kinetic energy. Assume laminar flow in the tube and (i) power-law behaviour and (ii) Bingham plastic behaviour.

3.10 Estimate the time needed to empty a cylindrical vessel (D_T = 101 mm) (open to atmosphere) filled with a power-law liquid (m = 4 Pa s^n and n = 0.6, density = 1010 kg/m^3). A 6-mm ID capillary tube 1.5-m long is fitted to the base of the vessel as shown in the diagram for Problem 3.9. The initial height of liquid in the vessel, H_0 = 230 mm. Estimate the viscosity of a hypothetical Newtonian fluid of the same density which would empty in the same time.

3.11 Using the same equipment as in Problem 3.10, the following height–time data have been obtained for a coal slurry of density 1135 kg/m^3. Evaluate the power-law model parameters for this slurry (Assume streamline flow under all conditions).

t (s)	0	380	808
H (m)	0.25	0.20	0.15

3.12 A pharmaceutical formulation having a consistency coefficient of 2.5 Pa s^n and a flow behaviour index of 0.65 must be pumped through a stainless steel pipe of 40-mm inside diameter. If the shear rate at the pipe wall must not exceed $140\,s^{-1}$ or fall below $50\,s^{-1}$, estimate the minimum and maximum acceptable volumetric flow rates.

3.30 For the laminar flow of a time-independent fluid between two parallel plates (Figure 3.18), derive a Rabinowitsch–Mooney type relation giving:

$$\left(-\frac{dV_z}{dy}\right)_{wall} = \left(\frac{3V}{h}\right)\left[\frac{2}{3} + \frac{1}{3}\frac{d\,ln(3V/h)}{d\,ln\,\tau_w}\right]$$

where $2h$ is the separation between two plates of width W ($\gg 2h$), τ_w is the wall shear stress. What is the corresponding shear rate at the wall for a Newtonian fluid?

3.31 A drilling fluid consisting of a china clay suspension of density $1090\,kg/m^3$ flows at $0.001\,m^3/s$ through the annular cross-section between two concentric cylinders of radii 50 and 25 mm, respectively. Estimate the pressure gradient if the suspension behaves as:

(i) a power-law liquid: $n = 0.3$ and $m = 9.6\,Pa\,s^n$.
(ii) a Bingham plastic fluid: $\mu_B = 0.212\,Pa\,s$ and $\tau_0^B = 17\,Pa$.

Use the rigorous methods described in as the approximate method presented . Assume streamline flow.

Due to pump malfunctioning, the available pressure gradient is only 75% of that calculated above. What will be the corresponding flow rates on the basis of the power-law and Bingham plastic models?

3.28 When a non-Newtonian liquid flows through a 7.5-mm diameter and 300-mm long straight tube at $0.25\,m^3/h$, the pressure drop is 1 kPa.

(i) Calculate the viscosity of a Newtonian fluid for which the pressure drop would be the same at that flow rate?
(ii) For the same non-Newtonian liquid, flowing at the rate of $0.36\,m^3/h$ through a 200-mm long tube of 7.5 mm in diameter, the pressure drop is 0.8 kPa. If the liquid exhibits power-law behaviour, calculate its flow behaviour index and consistency coefficient.
(iii) What would be the wall shear rates in the tube at flow rates of 0.18 and $0.36\,m^3/h$. Assume streamline flow.

3.15 A power-law liquid is flowing under streamline conditions through a horizontal tube of 8 mm diameter. If the mean velocity of flow is 1 m/s and the maximum velocity at the centre is 1.2 m/s, what is the value of the flow behaviour index?

For a Newtonian, organic liquid of viscosity 0.8 m Pas, flowing through the tube at the same mean velocity, the pressure drop is 10 kPa compared with 100 kPa for the power-law liquid. What is the power-law consistency coefficient, m, of the non-Newtonian liquid?

3.16 The rheology of a polymer solution can be approximated reasonably well by either a power-law or a Casson model over the shear rate range of $20–100\,s^{-1}$.

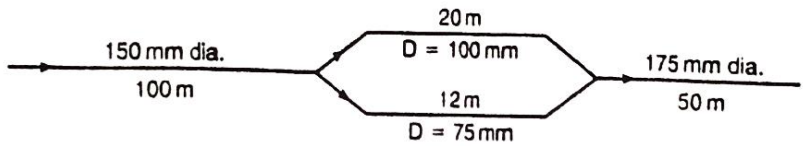
If the power law consistency coefficient, m, is 10 Pa s^n and the flow behaviour index, n, is 0.2, what will be the approximate values of the yield stress and the plastic viscosity in the Casson model?

Calculate the pressure drop using the power-law model when this polymer solution is in laminar flow in a pipe 200-m long and 40-mm inside diameter for a centreline velocity of 1 m/s. What will be the calculated centreline velocity at this pressure drop if the Casson model is used?

3.21 In a horizontal pipe network, a 150-mm diameter and 100-m long pipe branches out into two pipes, one 100-mm diameter and 20-m long and the other 75-mm diameter and 12-m long; the branches re-join into a 175-mm diameter and 50-m long pipe. The volumetric flow rate of a liquid (density 1020 kg/m^3) is 3.4 m^3/min and the pressure at the inlet to 150-mm diameter is 265 kPa. Calculate the flow rate in each of the two smaller diameter branches and the pressure at the beginning and end of the 175-mm diameter pipe for:

(a) a liquid exhibiting power-law behaviour, n = 0.4; m = 1.4 Pa s^n.
(b) a liquid exhibiting Bingham plastic behaviour, (yield stress 4.3 Pa and plastic viscosity 43 m Pa s).

Due to process modifications, the flow rate must be increased by 20%; Calculate the pressure now required at the inlet to the pipe network.



3.22 The fluid whose rheological data are given in Problem 3.19 is to be pumped at 0.34 m^3/s through a 380-mm diameter pipe over a distance of 175 m. What will be the required inlet pressure, and what pump power will be needed?

3.18 A viscous plastic fluid of density 1400 kg/m^3 and containing 60% (by weight) of a pigment is to be pumped in streamline flow through 75-, 100- and 125-mm diameter pipes within the plant area. The corresponding flow rates of the liquid in these pipes are 6×10^{-3}, 0.011 and 0.017 m^3/s, respectively. The following data were obtained on an extrusion rheometer using two different size capillaries:

Gas pressure in reservoir, $-\Delta p$ (kPa)	Time to collect 5 g liquid from the tube (s)	
	Tube A	Tube B
66.2	913	112
132.4	303	39
323.7	91	11.6
827.4	31	3.9

Tube A: diameter = 1.588 mm, length = 153 mm.
Tube B: diameter = 3.175 mm, length = 305 mm.
Estimate the pressure gradient in each of the pipes to be used to carry this liquid.

3.42 A bentonite slurry (of density 1400 kg/m^3) behaves like a Herschel–Bulkley fluid (m = 0.68 Pa s^n; n = 0.6; τ_0^H = 20 Pa) and it is to be heated in a 4-m long double pipe heat exchanger (R_i = 16.5 mm; R_o = 25.5 mm); Calculate the pressure drop when this slurry flows inside the tube and in the annular region when the flow rate is 3.41 m^3/h. Assume the flow to be laminar in both cases.