Stress and Strain

Since rheology is the study of the deformation of matter, it is essential to have a good understanding of stress and strain. Consider a rectangular bar that, due to a tensile force, is slightly elongated (Fig. 1.2). The initial length of the bar is L_o and the elongated length is L where $L = L_o + \delta L$ with δL representing the increase in length. This deformation may be thought of in terms of Cauchy strain (also called engineering strain):

$$\varepsilon_c = \frac{\delta L}{L_o} = \frac{L - L_o}{L_o} = \frac{L}{L_o} - 1 \qquad [1.1]$$

or Hencky strain (also called true strain) which is determined by evaluating an integral from L_o to L:

$$\varepsilon_h = \int_{L_o}^{L} \frac{dL}{L} = \ln(L/L_o) \qquad [1.2]$$

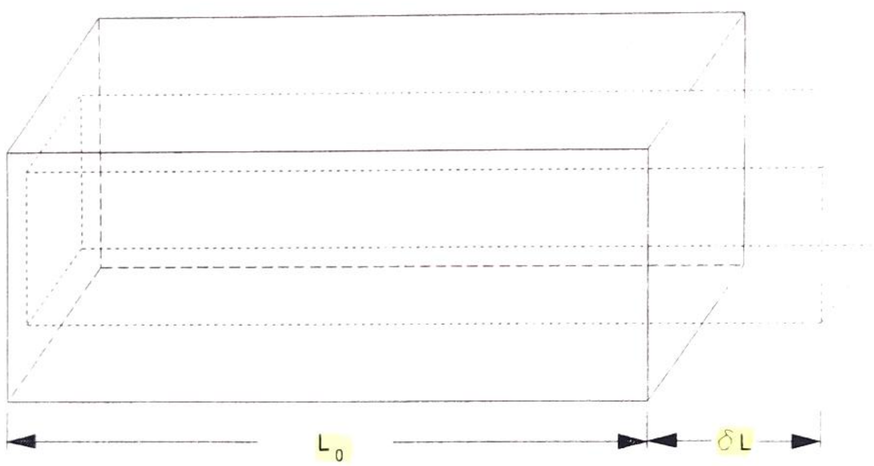

Figure 1.2. Linear extension of a rectangular bar.

Cauchy and Hencky strains are both zero when the material is unstrained and approximately equal at small strains. The choice of which strain measure to use is largely a matter of convenience and one can be calculated from the other:

$$\varepsilon_h = \ln(1 + \varepsilon_c) \qquad [1.3]$$

ε_h is preferred for calculating strain resulting from a large deformation.

Another type of deformation commonly found in rheology is simple shear. The idea can be illustrated with a rectangular bar (Fig. 1.3) of height h. The lower surface is stationary and the upper plate is linearly displaced by an amount equal to δL. Each element is subject to the same level of deformation so the size of the element is not relevant. The angle of shear, γ, may be calculated as

$$\tan(\gamma) = \frac{\delta L}{h} \tag{1.4}$$

With small deformations, the angle of shear (in radians) is equal to the shear strain (also symbolized by γ), $\tan\gamma = \gamma$.

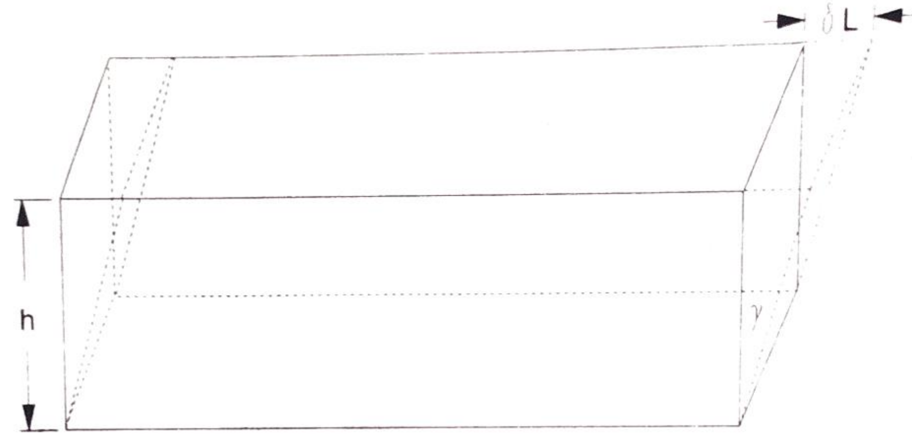

Figure 1.3. Shear deformation of a rectangular bar.

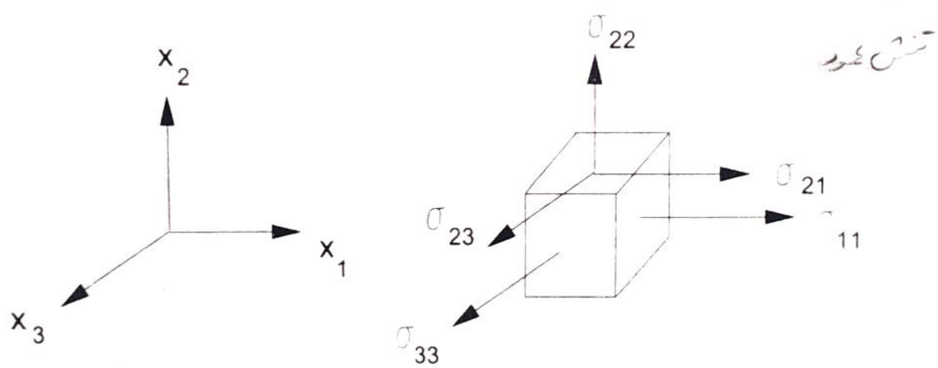

Figure 1.4. Typical stresses on a material element.

Stress, defined as a force per unit area and usually expressed in Pascal (N/m^2), may be tensile, compressive, or shear. Nine separate quantities are required to completely describe the state of stress in a material. A small element (Fig. 1.4) may be considered in terms of

Cartesian coordinates (x_1, x_2, x_3). Stress is indicated as σ_{ij} where the first subscript refers to the orientation of the face upon which the force acts and the second subscript refers to the direction of the force. Therefore, σ_{11} is a normal stress acting in the plane perpendicular to x_1 in the direction of x_1 and σ_{23} is a shear stress acting in the plane perpendicular to x_2 in the direction of x_3. Normal stresses are considered positive when they act outward (acting to create a tensile stress) and negative when they act inward (acting to create a compressive stress).

Stress components may be summarized as a stress tensor written in the form of a matrix:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad [1.5]$$

A related tensor for strain can also be expressed in matrix form. Basic laws of mechanics, considering the moment about the axis under equilibrium conditions, can be used to prove that the stress matrix is symmetrical:

$$\sigma_{ij} = \sigma_{ji} \qquad [1.6]$$

so

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$$\sigma_{12} = \sigma_{21} \qquad [1.7]$$

$$\sigma_{31} = \sigma_{13} \qquad [1.8]$$

$$\sigma_{32} = \sigma_{23} \qquad [1.9]$$

meaning there are only six independent components in the stress tensor represented by Eq. [1.5].

Equations that show the relationship between stress and strain are either called rheological equations of state or constitutive equations. In complex materials these equations may include other variables such as time, temperature, and pressure. A modulus is defined as the ratio of stress to strain while a compliance is defined as the ratio of strain to stress. The word rheogram refers to a graph of a rheological relationship.

Extensional Flow

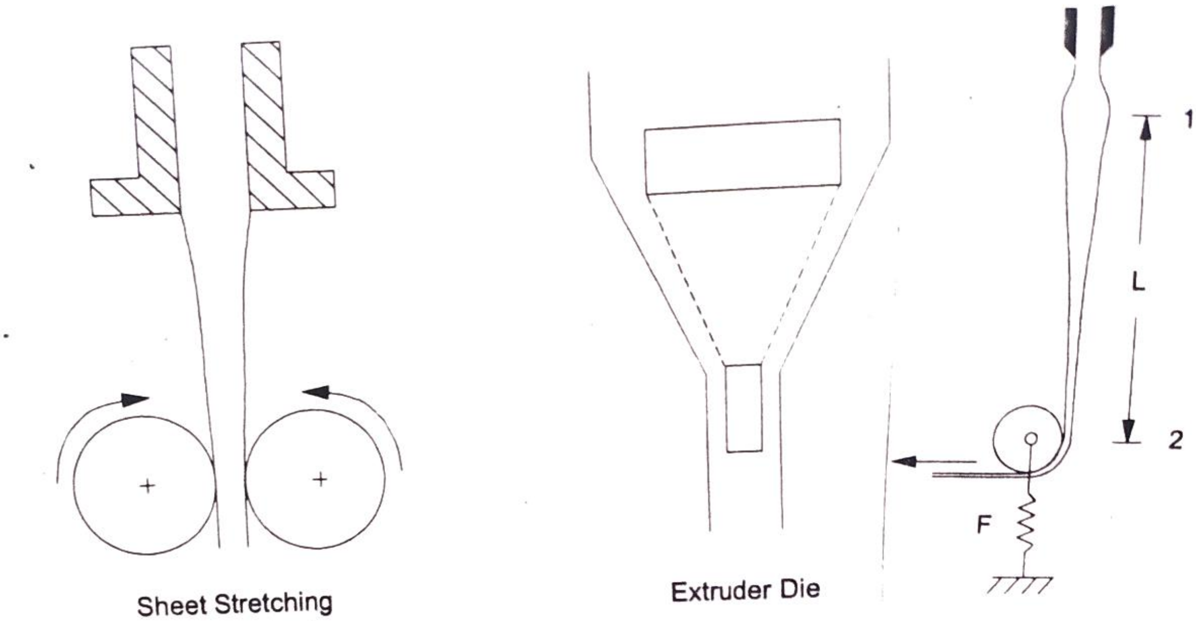

Figure 1.21. Extensional flow found in sheet stretching (or extrudate drawing) and convergence into an extruder die.

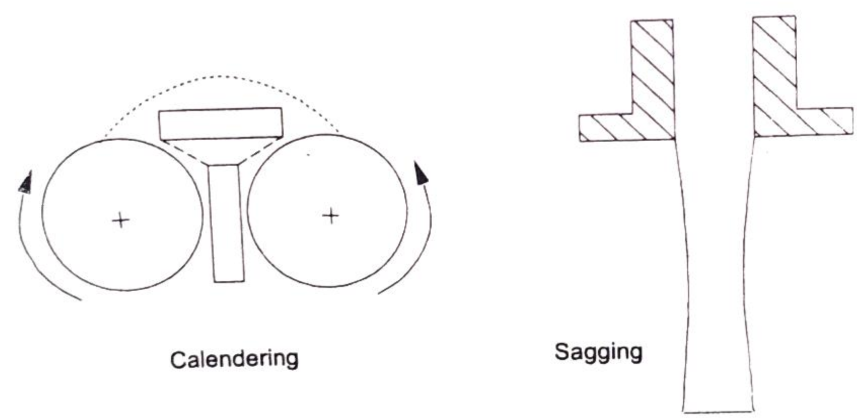

Figure 1.22. Extensional flow in calendering and gravity induced sagging.

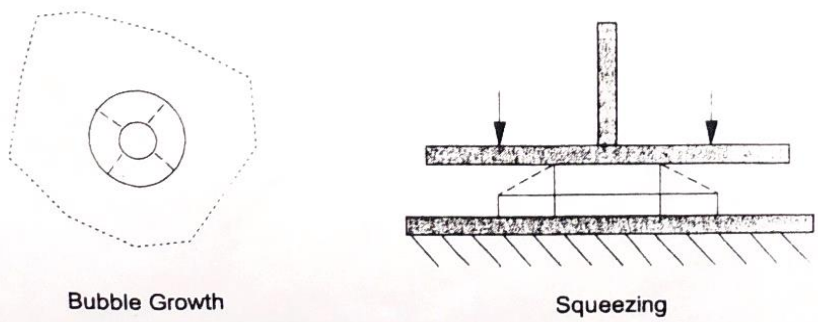

Figure 1.23. Extensional flow found in bubble growth and squeezing flow between lubricated plates.

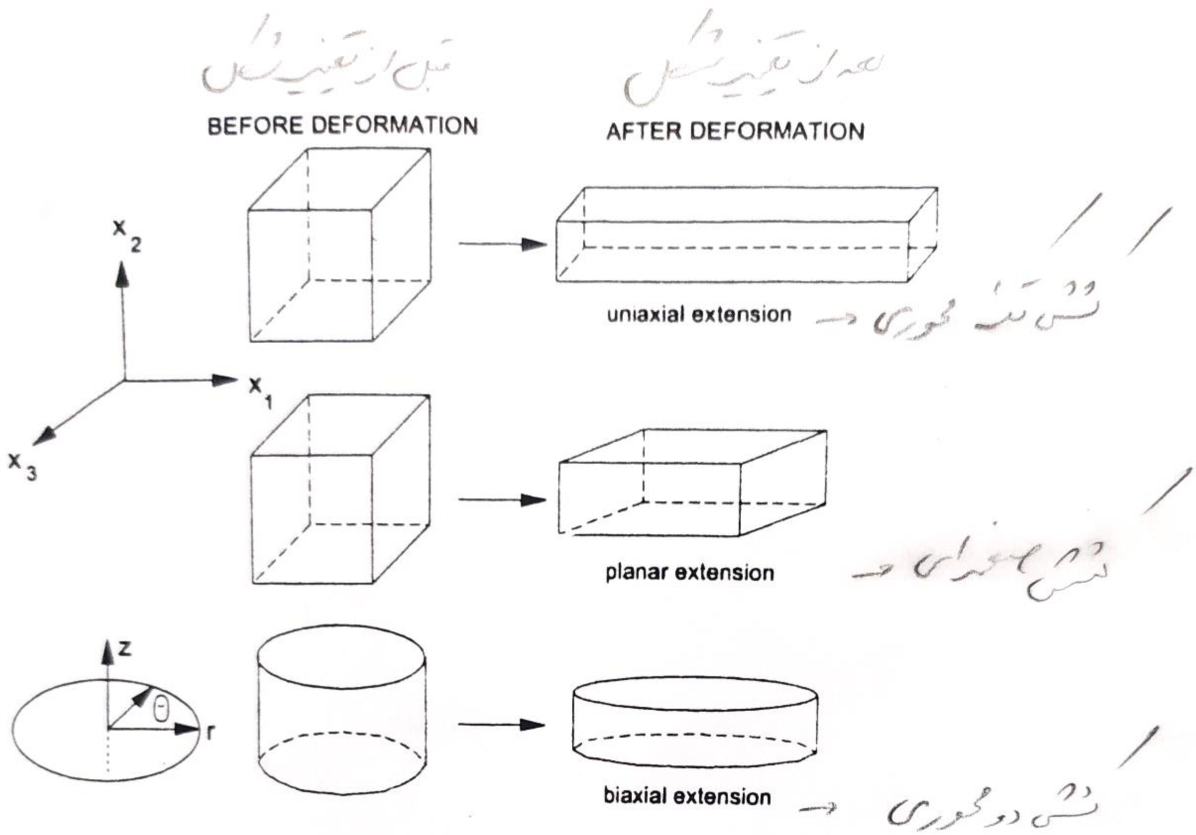

Figure 1.24. Uniaxial, planar, and biaxial extension.